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Development and Validation of a Mathematics Toolkit Integrating Mathematical Practice 8 (Repeated Reasoning) to Enhance Conceptual Understanding of Grade 10

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ABSTRACT

One persistent concern in mathematics education is the presence of least-learned competencies that hinder students' conceptual understanding and academic achievement. This study aimed to assess, design, develop, implement, and evaluate a Mathematics Toolkit integrating Mathematical Practice 8 (Repeated Reasoning) to enhance the conceptual understanding of Grade 10 learners at Lighthouse Charter School, Oakland, California, USA, during the School Year 2025–2026. The study involved 82 Grade 10 learners and was anchored on the ADDIE Model. A developmental research design employing descriptive developmental and qualitative approaches was utilized. Three researcher-made evaluation instruments were used to assess the toolkit in terms of content validity, instructional design and clarity, usability and student engagement, alignment to MP8, and overall quality. Quantitative data were analyzed using frequency count, mean, standard deviation, Shapiro-Wilk Test of Normality, Wilcoxon Signed-Rank Test, and Kruskal-Wallis Test, while qualitative data were analyzed through thematic analysis. The findings revealed that Geometry and Right Triangle concepts were the least-learned competencies among Grade 10 learners. The developed toolkit, which incorporated the MP8 routine of Notice → Repeat → Generalize, significantly improved learners' posttest performance and demonstrated a very large effect size. Validators strongly agreed that the toolkit met standards of quality, relevance, and alignment to MP8. The study highlights the importance of developmental research and structured instructional design in producing innovative learning resources that address learners' needs and strengthen conceptual understanding in mathematics.

Keywords: *Mathematics Toolkit, Mathematical Practice 8, Repeated Reasoning, ADDIE Model, Developmental Research, Conceptual Understanding*

1. INTRODUCTION

Mathematics education is fundamental in developing students' analytical thinking, reasoning abilities, problem-solving skills, and conceptual understanding necessary for academic achievement and real-life applications. In contemporary mathematics instruction, there is increasing emphasis on learner-centered approaches that promote deeper understanding rather than mere procedural knowledge. In line with this perspective, the present study focuses on the development and validation of a Mathematics Toolkit integrating Mathematical Practice 8 (Repeated Reasoning) intended for Grade 10 students. The toolkit was designed as an instructional intervention that encourages learners to recognize patterns, analyze mathematical structures, construct logical relationships, and apply repeated reasoning in solving mathematical problems. The study primarily delves into the process of designing, developing, validating, and evaluating the Mathematics Toolkit as a supplementary instructional material aligned with the Grade 10 Mathematics curriculum.

Reasoning occupies a central role in mathematics education, as it allows students to comprehend and overcome fundamental challenges through processes of logic, abstraction, and problem-solving. When learners engage in reasoning, they move beyond rote memorization and begin to recognize connections between concepts, thereby deepening their understanding. Scholars such as Odiri (2023), Mukuka (2020), and Supriadi and Suherman (2023) have emphasized that reasoning is indispensable for building a strong foundation in mathematics, enhancing problem-solving skills, and fostering motivation. Despite this, numerous studies reveal that many students continue to struggle with reasoning, underscoring the need for effective instructional strategies. This study focuses on the use of MP8—Repeated Reasoning—as a pedagogical approach to strengthen mathematical reasoning among Grade 10 learners.

The global challenge of mathematics proficiency further highlights the urgency of this issue. In California, only about one-third of students demonstrate proficiency in mathematics, with Grade 10 learners particularly struggling in reasoning and comprehension. UNESCO (2022) reported that more than 617 million students worldwide fail to achieve minimum proficiency levels in reading and mathematics, including 387 million primary school-aged children and 230 million adolescents. These figures reveal persistent achievement gaps and emphasize the importance of early mathematics instruction, as disparities often emerge before kindergarten (Blume & Watanabe, 2023). Contributing factors include weak foundational knowledge, poor problem-solving skills, reliance on memorization, difficulty with mathematical language, and limited use of metacognitive strategies (Gafoor, 2025; Mangarin & Caballes, 2024). Such challenges are compounded by anxiety, low motivation, and teaching practices that fail to make mathematics engaging or relevant.

This study contributes to mathematics education by providing an innovative instructional toolkit that integrates Mathematical Practice 8 (Repeated Reasoning) to strengthen conceptual understanding among Grade 10 students. The study expands existing literature on mathematical practices by demonstrating how repeated reasoning can be systematically embedded into instructional materials and classroom activities to improve students' ability to recognize patterns, formulate generalizations, and understand mathematical relationships. Moreover, the validation of the mathematics toolkit offers empirical evidence regarding its effectiveness, usability, and instructional value, which may serve as a basis for future curriculum development, teaching strategies, and educational interventions in mathematics education. The findings of the study may also guide teachers, curriculum developers, and researchers in designing learner-centered materials that promote deeper mathematical thinking and meaningful learning experiences.

1.1 Statement of the Problem

The study centered its investigation on assessing the proposed material in enhancing mathematical understanding of the Grade 10 students. Specifically, it sought to answer the following questions:

1. What are the least learned competencies of the students?
2. What design can be developed based on the least learned competencies?
3. What are the pretest-posttest results based on the field tryout?
4. Is there a significant difference between the pretest and posttest of the study field tryout?
5. What is the result of the content validation as expressed by the experts"?
   5.1 Mathematics Teachers;
   5.2 Local Content Experts and Foreign Content Experts.
6. What are the perceptions of the three groups of respondents on the developed material as to:
   6.1 Content,
   6.2 Format,
   6.3 Presentation and
   6.4 Accuracy and up-to-datedness of the information
7. Is there a significant difference in the validation rating as perceived by the three groups of respondents?
8. What are prevailing challenges in the utilization of the proposed material?
9. What enhanced output can be proposed based on the findings of the study?

1.2 Literature Review

Mathematical reasoning

It plays a vital role in developing students' problem-solving skills, logical thinking, and conceptual understanding. According to Kaitera (2022), reasoning provides a structured approach to analyzing and solving complex problems, while Putra and Sarigar (2024) emphasized that it helps learners understand abstract concepts and fosters creativity. Similarly, Sabbagh and Al-Khayat (2020) explained that reasoning enables students to comprehend mathematical concepts logically and formulate conclusions and generalizations. Oz (2024) highlighted that reasoning-based instruction promotes mathematical understanding through problem solving, inquiry, and discussion. Moreover, Palengka (2022) stressed that students should be given opportunities to solve non-routine problems and develop creative and justifiable reasoning. Marrasabesy (2021) found that challenging learning activities improve students' abilities in generalization and justification, while Stanford (2022) described reasoning as the ability to think logically and apply prior knowledge to unfamiliar situations.

Furthermore, Bayat and Noroozi (2022) revealed that reasoning strategies improve students' decision-making skills and learning satisfaction. Likewise, Oco (2023) reported that reasoning skills are significantly associated with mathematics performance. In addition, Mukuka (2023) and Mukuka et al. (2025) emphasized that mathematical reasoning develops critical thinking, conceptual understanding, and real-world application of mathematics. Correspondingly, Ponte (2022) asserted that reasoning enables students to understand mathematical concepts as logical and interconnected ideas, while Tum (2020) emphasized that reasoning strategies enhance information processing, problem-solving, and everyday application of mathematical concepts. Finally, the ADDIE Model, as discussed by Li and Cheong (2023) and Spatioti, Kazanidis, and Pange (2022), provides a systematic framework for developing and evaluating instructional materials. These studies collectively support the development and validation of a Mathematics Toolkit integrating Mathematical Practice 8 (Repeated Reasoning) to enhance the conceptual understanding of Grade 10 students.

Challenges in Implementing Mathematics Instructional Materials

The implementation of mathematics instructional materials may encounter several challenges that affect students' learning and participation. These include cognitive and problem-solving challenges, where students struggle to apply prior knowledge to unfamiliar situations and often rely on procedural skills rather than conceptual understanding (Schoenfeld, 2023). Affective challenges, such as mathematics anxiety and low self-confidence, may also hinder students' concentration, participation, and performance in mathematics (Ashcraft, 2002). In addition, motivational challenges influence students' willingness to engage actively in learning activities, with learners showing greater persistence when lessons are meaningful and relevant (Deci & Ryan, 2000). Furthermore, individual differences in cognitive abilities, learning styles, prior knowledge, and reasoning skills require differentiated instruction to ensure that all learners have equal opportunities to succeed (Tomlinson, 2023). These challenges highlight the importance of designing instructional materials that are engaging, learner-centered, and responsive to students' diverse needs.

Synthesis

Mathematical reasoning is an essential component of problem-solving that enables students to think logically, analyze patterns, and develop conceptual understanding. Studies by Kaitera (2022), Putra and Sarigar (2024), Sabbagh and Al-Khayat (2020), and Oz (2024) emphasized that reasoning promotes higher-order thinking, meaningful learning, and mathematical sense-making. Likewise, Palengka (2022), Marrasabesy (2021), Stanford (2022), Bayat and Noroozi (2022), Oco (2023), Mukuka (2023), Mukuka et al. (2025), Ponte (2022), and Tum (2020) highlighted that reasoning-based instruction improves students' critical thinking, decision-making, problem-solving skills, and conceptual understanding. Furthermore, the ADDIE Model, as discussed by Li and Cheong (2023) and Spatioti, Kazanidis, and Pange (2022), provides a systematic framework for developing effective instructional materials. The literature also emphasizes the importance of evaluating instructional materials in terms of content validity, instructional design, usability, student engagement, and alignment with educational standards.

2. Methods

The study employed a Developmental–Evaluative Research Design using the ADDIE Model (Analysis, Design, Development, Implementation, and Evaluation) to develop and validate a Mathematics Toolkit integrating Mathematical Practice 8 (Repeated Reasoning). The study was conducted at Lighthouse Community Public School in Oakland, California, with 75 Grade 10 students, 4 local content experts, and 3 foreign content experts, selected through purposive sampling.

Three researcher-made instruments were utilized: the Evaluation Questionnaire, Validation Questionnaire, and Perception and Challenges Questionnaire. These instruments underwent expert validation and reliability testing, yielding a Cronbach's Alpha of 0.73, indicating high reliability.
Data gathering involved securing permissions, developing the toolkit, conducting expert validation, administering the instruments, and collecting qualitative and quantitative data. Ethical principles such as informed consent, confidentiality, data privacy, beneficence, and justice were strictly observed throughout the study. Data were analyzed using Mean, Wilcoxon Signed-Rank Test, and Kruskal-Wallis H Test to determine the toolkit's validity, effectiveness, and acceptability in enhancing the conceptual understanding of Grade 10 students.

3. Results and Discussion

Table 3.1 Least learned competencies of the students
Table 1 presents the least learned competencies of the learners.

Least Learned Competency	Frequency (n=75)	Percentage	Description
Verify experimentally the properties of dilations	61	81.3%	Students struggled in verifying dilation properties and identifying scale factors using geometric figures.
Use the definition of similarity/transformations to determine if two figures are similar	58	77.3%	Students had difficulty identifying similar figures using transformations and similarity rules.
Prove theorems about triangles	54	72%	Students struggled in proving geometric relationships and triangle properties logically.
Use congruence/similarity criteria to solve problems	50	66.7%	Students experienced challenges in applying congruence and similarity criteria in problem-solving situations.
Understand that side ratios in right triangles are properties of the angles	46	61.3%	Students showed weak understanding of trigonometric ratios and angle-side relationships.
Explain the relationship between sine and cosine of complementary angles	42	56.0%	Students struggled to connect sine and cosine relationships conceptually.

The identified least-learned competencies reflect persistent conceptual and procedural gaps in students' understanding of geometric transformations, similarity, triangle reasoning, and trigonometry. Recent research in mathematics education consistently shows that these difficulties are rooted in weak conceptual foundations, limited visualization skills, and insufficient connections between algebraic, geometric, and functional representations.

The table 3.1 presents the least-learned competencies in High School Geometry under the CCSS framework, together with descriptions of the specific difficulties encountered by students. The identified least-learned competencies include verifying experimentally the properties of dilations, using definitions of similarity and transformations to determine whether two figures are similar, proving theorems about triangles, applying congruence and similarity criteria in problem-solving, understanding side ratios in right triangles as properties of angles, and explaining the relationship between sine and cosine of complementary angles.

Table 3.2 Pretest-posttest Results based on the Field Tryout
Table 2: Field Tryout Results of the Pre-Test and Post-Test

	Field Tryout			
	Pre-Test	SD	Post Test	SD
Mean Scores	54.83	4.17	81.46	3.27

The findings of the field tryout indicate a substantial improvement in the academic performance and conceptual understanding of the respondents after exposure to the developed Mathematics Toolkit. The pre-test results suggest that prior to the implementation of the intervention, many students

experienced difficulties in comprehending and applying the targeted mathematical concepts and reasoning skills. Such findings imply that learners initially possessed limited mastery of the competencies covered in the lessons, particularly in areas requiring conceptual analysis, pattern recognition, and repeated reasoning.

Table 3.3 Significant Difference between the Pretest and Posttest of the Study Field Tryout
Table 3: T-test to Compare the Pre-test and Post-test Results on the use of Repeated Reasoning

Source of Variation		Statistic	P	Decision	Conclusion
Pretest	Posttest	200*	<.001	Reject H_0	Sig

Legend Interpretation: $p < 0.05$-Significant (Sig), $p \geq 0.05$-Not Significant (NS), Reject H_0-There is a significant difference between the pre-test and post-test scores, Accept H_0-There is no significant difference between the pre-test and post-test scores.

The table 3 below presents the results of the statistical analysis conducted to determine the significant difference between the pre-test and post-test performances of the respondents after the implementation of the Mathematics Toolkit integrating Mathematical Practice 8 (Repeated Reasoning). The Wilcoxon Signed-Rank Test was utilized because the data violated the assumption of normality based on the Shapiro–Wilk test. This nonparametric statistical procedure was deemed appropriate in determining whether the instructional intervention produced significant changes in students' mathematical performance and conceptual understanding. The findings of the study revealed a highly significant difference between the pre-test and post-test performances of the respondents following the implementation of the Mathematics Toolkit integrating repeated reasoning strategies. The significant result indicates that the instructional intervention substantially improved students' conceptual understanding, procedural fluency, and problem-solving abilities in Geometry. The rejection of the null hypothesis confirms that the improvement observed in students' performance was not merely due to chance or random variation, but rather a direct effect of the implemented instructional material and learning strategies. The results imply that the Mathematics Toolkit effectively addressed the identified learning gaps and difficulties encountered by students in understanding geometric concepts and mathematical relationships.

3.4 Content Validation as Expressed by the Experts
Table 4 Summary Results on the Validation of Mathematics Toolkit

Criteria	Math teacher		Local expert		Foreign expert	
	Mean	Description	Mean	Description	Mean	Description
Content Validity	4.90	SA	4.40	SA	4.93	SA
Instructional Design and Clarity	5.00	SA	4.30	SA	4.87	SA
Usability and Student Engagement	4.70	SA	4.40	SA	4.80	SA
Alignment to Mathematical Practice 8 (MP8 – Repeated Reasoning)	5.00	SA	4.50	SA	5.00	SA
Overall Quality	4.80	SA	5.00	SA	4.80	SA

Legend: (5)4.21 – 5.00-Strongly Acceptable (SA), (4) 3.41 – 4.20-Acceptable (A), (3)2.61 – 3.40-Moderately Acceptable (MA), 2(1.81 – 2.60)-Less Acceptable (LA), 1(1.00 – 1.80) Not Acceptable (NA)

The table 4 presents the summary results of the validation conducted on the developed Mathematics Toolkit integrating Mathematical Practice 8 (Repeated Reasoning) as evaluated by Mathematics teachers, local experts, and foreign experts. The evaluation focused on essential dimensions of instructional material quality, namely content validity, instructional design and clarity, usability and student engagement, alignment to Mathematical Practice 8 (MP8 – Repeated Reasoning), and overall quality. The purpose of the validation process was to determine the acceptability, instructional soundness, and pedagogical effectiveness of the developed toolkit for Grade 10 Mathematics instruction.

The findings of the validation revealed that the Mathematics Toolkit was highly acceptable and demonstrated strong instructional, pedagogical, and content-related qualities necessary for effective Mathematics instruction. The consistently favorable evaluations from Mathematics teachers, local experts, and foreign experts indicate that the developed instructional material successfully met the standards expected of quality educational resources. The strong validation results imply that the toolkit was carefully designed to support conceptual understanding, mathematical reasoning, learner engagement, and mastery of competencies aligned with the Grade 10 Mathematics curriculum.

The high validation results in terms of content validity suggest that the lessons, examples, exercises, and assessment activities included in the toolkit were relevant, accurate, curriculum-aligned, and responsive to the learning needs of students. The evaluators recognized that the instructional material provided meaningful opportunities for learners to explore mathematical concepts through repeated reasoning and conceptual analysis rather than through rote memorization alone. This implies that the toolkit effectively addressed the least learned competencies and facilitated the development of deeper mathematical understanding among learners. The findings further indicate that the instructional material was grounded on sound mathematical content and appropriate learning progressions that supported mastery learning and meaningful engagement with Geometry concepts.

3.5 Perceptions of the Groups of Respondents on the Developed Material

Table 5 Validators Perceptions on Developed Material

Criteria	Math teacher		Local expert		Foreign expert	
	Mean	Description	Mean	Description	Mean	Description
Content	4.90	SA	4.50	SA	4.87	SA
Format	4.60	SA	4.30	SA	4.60	SA
Presentation and Accuracy	4.80	SA	4.30	SA	4.60	SA
Up-to-datedness of the information	5.00	SA	4.50	SA	4.80	SA

Legend:4.21 – 5.00 = Strongly Acceptable (SA), 3.41 – 4.20 = Acceptable (A), 2.61 – 3.40 = Moderately, Acceptable (MA), 1.81 – 2.60 = Less Acceptable (LA), 1.00 – 1.80 = Not Acceptable (NA)

3.6 Significant Difference in the Validation rating as Perceived by the Three Groups of Respondents

Table 6 Results on the Evaluation of the Mathematics Toolkit

Criteria	H-Test	P-value	Conclusion
Content	6.622	0.036	Significant
Format	4.427	0.109	Not Significant
Presentation	11.594	0.003	Significant
Up-to-datedness	11.170	0.004	Significant

Legend:p-value < 0.05 = Significant (S), p-value ≥ 0.05 = Not Significant (NS)
Decision Rule: Reject H_0 if p-value < 0.05, Accept H_0 if p-value ≥ 0.05

The table below presents the results of the Kruskal–Wallis Test conducted to determine whether there was a significant difference in the perceptions of the three groups of respondents—Mathematics teachers, local experts, and foreign experts—regarding the developed Mathematics Toolkit integrating Mathematical Practice 8 (Repeated Reasoning). The test was employed to compare the evaluators' responses across the criteria of content, format, presentation, and accuracy/up-to-dateness of information. Since the data involved ordinal ratings from multiple independent groups, the Kruskal–Wallis Test was deemed appropriate to determine whether variations existed in the evaluators' assessments of the instructional material.

The findings revealed that significant differences existed among the perceptions of the three groups of validators in terms of content, presentation, and accuracy or up-to-datedness of information. These results imply that the Mathematics teachers, local experts, and foreign experts differed in their evaluation of certain instructional dimensions of the developed toolkit. The significant variation in content evaluation suggests that the validators may have assessed the relevance, depth, curriculum alignment, and organization of mathematical concepts from different professional and contextual perspectives. Mathematics teachers likely emphasized classroom applicability and learner responsiveness, while local and foreign experts may have focused more on academic rigor, curriculum standards, and international pedagogical expectations.

3.7 Prevailing Challenges for Teachers in the Use of the Material

Table 6 Prevailing Challenges of Teachers

Theme	Sub-Themes
Instructional Facilitation Challenges	Guiding students through the problem-solving process without directly giving answers
	Difficulty facilitating Mathematical Practice 8 (Repeated Reasoning) effectively
Assessment and Evaluation Challenges	Difficulty evaluating students' problem-solving processes and perseverance
	Challenges in assessing students' conceptual understanding and reasoning
Time Management Challenges	Limited instructional time during mathematics lessons
	Difficulty balancing curriculum coverage and reasoning-based instruction
Professional Development Needs	Need for continuous training on problem-solving strategies
	Need for professional development in assessing Mathematical Practice 8
Classroom Management and Learner Behavior Challenges	Difficulty understanding students' behavior during mathematics activities
	Managing students with varying learning styles and engagement levels

The data presented in Table 8 reveals several prevailing challenges encountered by teachers in the use of the Mathematics Toolkit integrating Mathematical Practice 8 (Repeated Reasoning). These challenges are categorized into themes related to instructional facilitation, assessment and evaluation,

time management, professional development, and classroom management. The findings suggest that while the toolkit promotes conceptual understanding and reasoning-centered learning, teachers still encounter practical and pedagogical difficulties in effectively implementing the material within actual classroom settings.

Theme 1: Instructional Facilitation Challenges

One major theme identified is Instructional Facilitation Challenges, particularly in guiding students through the problem-solving process without directly providing answers. Teachers reported difficulty facilitating Mathematical Practice 8 effectively because students often require continuous guidance in recognizing repeated reasoning, identifying patterns, and making mathematical generalizations independently.

> *"Students initially ask for direct answers, but with guiding questions, I noticed that they gradually become more confident in finding patterns and developing their own solutions." (T1)*
> *"Although students need support at first, continuous guidance helps them become more active and engaged in the learning process." (T2)*

The responses from both teachers and students suggest that while facilitating repeated reasoning initially required additional guidance and support, the process contributed positively to students' development of independent thinking and problem-solving skills. Teachers viewed the experience as an opportunity to strengthen facilitation strategies, while students recognized improvements in their confidence and mathematical understanding

Theme 2: Assessment and Evaluation Challenges

Another significant theme is Assessment and Evaluation Challenges. Teachers expressed difficulty in evaluating students' problem-solving processes, perseverance, and conceptual understanding. Unlike traditional mathematics instruction that focuses mainly on final answers, Mathematical Practice 8 requires teachers to assess how students analyze patterns, justify solutions, and apply reasoning strategies. This makes assessment more complex because teachers need to observe students' thinking processes, reasoning approaches, and persistence during problem-solving activities. The findings suggest that educators may require more structured assessment tools, rubrics, and observation guides to effectively evaluate higher-order mathematical reasoning and conceptual understanding.

> *"Assessing students through repeated reasoning activities helped me understand how they arrived at their answers instead of simply checking if the answers were correct." (T1)*
> *"Evaluating students' explanations and solution strategies required more observation, but it gave me a clearer picture of their conceptual understanding and reasoning abilities." (T2)*

Theme 3: Time Management Challenges

The findings also highlight Time Management Challenges, particularly the limited instructional time available during mathematics lessons. Teachers reported difficulty balancing curriculum coverage with reasoning-based instruction because implementing Mathematical Practice 8 often requires extended discussions, exploration, and reflection activities. This challenge indicates that reasoning-centered learning may demand more classroom time compared to conventional lecture-based teaching approaches. Teachers may struggle to complete required competencies while simultaneously ensuring that students engage deeply in repeated reasoning and conceptual exploration. The result underscores the need for efficient lesson planning and flexible instructional pacing when integrating innovative mathematics practices. Teachers stated that effective lesson planning and flexible instructional pacing may help

teachers balance curriculum requirements while promoting students' engagement in repeated reasoning and conceptual learning.

> *"Repeated reasoning activities required more time because students needed opportunities to discuss patterns and explain their thinking, but I noticed that these discussions strengthened their understanding of the lesson." (T1)*
> *"Balancing lesson coverage and reasoning activities was challenging at times, yet allowing students to explore concepts deeply helped them develop a better understanding rather than simply memorizing procedures." (T2)*

Theme 4: Professional Development Needs

Another important theme identified is Professional Development Needs. Teachers emphasized the need for continuous training on problem-solving strategies and assessment techniques related to Mathematical Practice 8. The findings suggest that some teachers may not yet feel fully prepared to implement reasoning-centered instruction effectively, particularly in facilitating open-ended problem-solving and evaluating students' reasoning processes. This highlights the importance of sustained professional development programs that equip teachers with modern pedagogical approaches, classroom facilitation techniques, and assessment strategies aligned with contemporary mathematics education standards. Continuous training is essential to strengthen teachers' confidence, competence, and readiness in implementing innovative instructional materials such as the Mathematics Toolkit. Teachers highlighted that students demonstrate different approaches and thought processes when solving mathematical problems, indicating the need for teachers to strengthen their facilitation skills through effective questioning and guided discussions.

> *"Using repeated reasoning activities made me realize that students think differently when solving problems. I feel that additional workshops on questioning techniques and facilitating discussions would help me guide students more effectively." (T1)*

This implies that facilitating Mathematical Practice 8 extends beyond content delivery and requires teachers to support students in constructing understanding through inquiry and reflection.

> *"The Mathematics Toolkit encouraged students to explain their thinking, but I think more training on assessing reasoning and conceptual understanding would strengthen my confidence in implementing these activities." (T2)*

These findings imply that sustained professional development programs may enhance teachers' competence, confidence, and readiness in implementing innovative instructional materials and learner-centered strategies that support meaningful mathematical understanding.

Theme 5: Classroom Management and Learner Behavior Challenges

Lastly, the findings reveal Classroom Management and Learner Behavior Challenges. Teachers reported difficulty understanding students' behavior during mathematics activities and managing learners with varying learning styles, engagement levels, and reasoning abilities. Since students differ in their mathematical background, confidence, and problem-solving skills, teachers may encounter challenges in maintaining active participation and ensuring that all learners benefit equally from reasoning-based tasks. Teachers recognized the importance of adapting instructional approaches to address diverse learning needs and maintain active student participation.

"I noticed that students learn and solve problems at different paces during repeated reasoning activities. Some students quickly identify patterns, while others need additional guidance and prompting to stay engaged in the discussion." (T1)
"Managing classroom activities became more dynamic because students demonstrated different levels of participation and confidence. However, providing support and encouraging collaboration helped students become more involved in the learning process." (T2)

Providing appropriate guidance and a supportive classroom environment helped increase student engagement and improve mathematical understanding among Grade 10 learners. Findings indicate that while the Mathematics Toolkit integrating Mathematical Practice 8 offers substantial instructional benefits, its implementation also presents significant challenges for teachers. The identified themes suggest that successful implementation requires strong facilitation skills, effective assessment practices, efficient time management, continuous professional development, and responsive classroom management strategies. Addressing these challenges is essential to maximize the effectiveness of the toolkit and ensure meaningful learning experiences for students in mathematics.

3.8 Prevailing Challenges for Students in the Use of the Material

Table 7 Prevailing Challenges for Students in the Use of the Material

Theme	Sub-Themes
Cognitive and Problem-Solving Challenges	Difficulty applying previously learned concepts to new situations
	Low problem-solving ability
Affective Challenges	Difficulty understanding complex mathematical concepts
	Mathematics anxiety
Motivational Challenges	Lack of confidence in solving mathematical problems
	Lack of interest in mathematics
	Low engagement during mathematics activities
Individual Differences in Learning	Varying levels of understanding among students
	Differences in learning pace and reasoning skills

Theme 1: Cognitive and Problem-Solving Challenges

One of the major themes identified is Cognitive and Problem-Solving Challenges. Under this theme, students demonstrated difficulty applying previously learned concepts to new problem-solving situations, low problem-solving ability, and difficulty understanding complex mathematical concepts. These findings suggest that many learners struggle with transferring prior knowledge to unfamiliar contexts, particularly in tasks requiring higher-order thinking and reasoning. Students may understand mathematical procedures in isolated situations but experience difficulty when concepts are integrated into more complex or non-routine problems.

The challenge also reflects limitations in conceptual understanding and analytical thinking, which are essential in geometry and reasoning-centered instruction. Since Mathematical Practice 8 emphasizes identifying repeated reasoning and recognizing mathematical structures, learners who have weak foundational knowledge may find it difficult to generalize patterns and formulate solutions independently.

Learners stated that they require repeated opportunities to engage in pattern recognition, reasoning activities, and concept application to strengthen mathematical understanding and problem-solving skills.

> *"I understood the examples during the lesson, but when the problem changed or looked different, I became confused about which concept I should apply." (S1)*
> *"The repeated reasoning activities helped me recognize patterns, but at first I found it difficult to connect previous lessons to new problems." (S2)*

The findings revealed that students experienced difficulties in applying previously learned concepts to unfamiliar mathematical situations, particularly in tasks requiring higher-order thinking and repeated reasoning. Learners showed challenges in transferring knowledge, recognizing patterns, and connecting concepts when solving complex problems. These findings suggest that students require continuous opportunities for guided practice and reasoning-based activities to strengthen conceptual understanding and improve problem-solving skills.

Theme 2: Affective Challenges

Another important theme is Affective Challenges, which include mathematics anxiety and lack of confidence in solving mathematical problems. The findings indicate that emotional factors significantly influence students' engagement and performance in mathematics. Mathematics anxiety may cause learners to feel nervous, pressured, or fearful when solving problems, reducing their concentration and willingness to participate in classroom activities. Likewise, lack of confidence may discourage students from attempting challenging tasks or expressing their reasoning during discussions.

The responses indicate that emotional factors such as anxiety and low confidence influenced students' participation and engagement during mathematics activities. Students initially showed hesitation in expressing ideas and solving unfamiliar problems because of fear of making mistakes. However, continuous participation in repeated reasoning activities gradually encouraged students to become more comfortable and confident in sharing their thinking processes.

> *"I felt nervous when solving problems in front of the class because I was afraid of making mistakes and giving the wrong answer." (S1)*
> *"At first, I was not confident in explaining my reasoning because I thought my answer might be incorrect, but after participating in repeated activities, I became more comfortable sharing my ideas." (S2)*

These findings support the theme of Affective Challenges, suggesting that positive reinforcement, supportive learning environments, and encouragement may help strengthen students' confidence and persistence in mathematics learning. These affective barriers can negatively impact students' persistence and problem-solving behavior, particularly in reasoning-based learning environments where active participation and exploration are expected. The findings imply that students may require emotional support, positive reinforcement, and anxiety-reduction strategies to build confidence and resilience in mathematics learning.

Theme 3: Motivational Challenges

The theme on Motivational Challenges further highlights students' lack of interest in mathematics and low engagement during mathematics activities. This suggests that some learners may perceive mathematics as difficult, abstract, or irrelevant to their daily experiences, resulting in reduced participation and limited enthusiasm toward learning tasks. Low engagement may also arise when students experience repeated difficulties in understanding lessons or solving problems, causing frustration

and decreased motivation. Since Mathematical Practice 8 requires active exploration, reasoning, and persistence, motivation becomes an important factor in sustaining learners' participation in mathematical activities.

Students' motivation was influenced by their level of understanding and confidence during repeated reasoning activities. Learners initially showed less interest when they encountered difficulties; however, meaningful examples, pattern recognition, and guided activities increased participation and engagement.

> *"At first, I thought the activities were difficult because I did not immediately understand the patterns, so I felt less interested in answering." (S1)*
> *"I became more interested when I realized that I could identify patterns and solve problems step by step because it made me feel that I could actually do the activity." (S2)*

The findings emphasize the importance of designing meaningful, interactive, and contextualized learning experiences that can stimulate students' curiosity and encourage active involvement in the learning process. Students' motivation was influenced by their understanding of the lesson, confidence level, and classroom experiences. Interactive and reasoning-based activities increased participation and interest, while teacher support and meaningful learning tasks encouraged students to become more engaged in mathematics learning.

Theme 4: Individual Differences in Learning

Another significant theme identified is Individual Differences in Learning. Students were found to have varying levels of understanding, differences in learning pace, and differences in reasoning skills. These findings indicate that learners possess diverse cognitive abilities, prior knowledge, and learning preferences, which affect how they respond to reasoning-centered instruction. Teachers stated that some students may quickly grasp mathematical relationships and patterns, while others require additional explanations, scaffolding, and practice opportunities.

Variations in learning pace and reasoning ability create challenges in classroom instruction because teachers must balance the needs of fast learners and struggling students simultaneously. This highlights the importance of differentiated instruction, flexible learning activities, and adaptive teaching strategies to ensure that all students can participate meaningfully and progress according to their individual learning needs.

Students' challenges in using the Mathematics Toolkit are not limited to academic difficulties alone but also involve emotional, motivational, and individual learning factors. The identified themes demonstrate that effective implementation of Mathematical Practice 8 requires instructional approaches that address both cognitive development and learner well-being.

> *"I noticed that some of my classmates understood the patterns faster than I did, so I needed more examples and practice before I understood the lesson completely." (S1)*
> *"The repeated reasoning activities helped me understand the lesson, but sometimes I needed more time and teacher guidance to explain my thinking and solve the problems." (S2)*

Providing scaffolded activities, engaging learning experiences, emotional support, and differentiated instruction may help reduce these challenges and improve students' conceptual understanding, confidence, engagement, and problem-solving abilities in mathematics.

4. CONCLUSIONS

Based on the results, the following conclusions were derived:

Mathematics Toolkit integrating Mathematical Practice 8 (Repeated Reasoning) was an effective instructional intervention in improving the conceptual understanding, reasoning ability, and problem-solving performance of Grade 10 students in geometry. The toolkit successfully addressed the identified least-learned competencies in transformations, similarity, congruence, proof construction, and trigonometric relationships through learner-centered, visualization-based, and reasoning-focused instructional strategies.

The significant improvement between the pre-test and post-test scores confirmed that the Mathematics Toolkit had a positive and meaningful effect on students' mathematics achievement. The results further demonstrated that repeated reasoning activities enhanced students' ability to analyze patterns, justify solutions, and apply mathematical concepts in various contexts.

The expert validators and respondents strongly agreed on the quality, validity, usability, and instructional effectiveness of the toolkit, indicating that the material is appropriate, reliable, and aligned with current mathematics education standards. Although differences existed in the evaluation ratings among validator groups, the overall findings consistently supported the acceptability and educational value of the toolkit.

The study also revealed that teachers and students encountered challenges in implementing and using the material, particularly in reasoning facilitation, assessment practices, time management, mathematics anxiety, motivation, and individual learning differences. These challenges suggest that effective implementation of reasoning-centered instructional materials requires adequate teacher preparation, continuous professional development, learner support, and differentiated instructional strategies. Overall, the Mathematics Toolkit integrating repeated reasoning has strong potential to enhance mathematics instruction and improve students' conceptual understanding and mathematical thinking skills.

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